that the values of μ are more accordant than might have been expected, having regard to accidental error only.

On analysing the last column but one for argument ☊ I find

(tabular *minus* observed) $(+0''{\cdot}40 \sin ☊ - 0''{\cdot}12 \cos ☊) \sin D$

The former term is far too large to be accidental; but as sin ☊ sin D splits into cosines, and not sines, it is probable that ☊ is not the actual argument of the periodicity, but only a near approximation to it. The point is accordingly held over till the Airy period is reduced. As far as I can foresee, this reduction should be complete in about two months.

There is a point to which attention should be called, although I am at present unable to offer an explanation. The *apparent* coefficient of sin D and the *apparent* correction to semi-diameter are naturally of the same sign as a rule; in fact, they are never of unlike sign. One test of the validity of the solution into true corrections is that there shall be no periodicity in the correction to semi-diameter, but that all the periodicity should belong to the correction to parallactic inequality. The results stand this test. There is, however, another test, viz. that the signs should be independent. Now in the forty-eight periods we have thirty-two cases of unlike sign, six of like sign, and ten cases where one at least of the quantities is zero. This test is therefore not satisfied. As a similarity of sign has been replaced by, on the whole, a dissimilarity, I conclude that the process of separation has been overdone.

Note on Elliptic Motion. By Asaph Hall.

The difference of the ratio of the radius vector and the mean distance from unity and the difference of the true and mean anomalies, or the equation of centre, are two important quantities in the theory of this motion. If these quantities are wanted in explicit terms of the eccentricity and the mean anomaly, perhaps the easiest method is to apply the theorem of Lagrange and to use the equation

$$dv = \sqrt{1-e^2} \,.\, \frac{a^2}{r^2} \,.\, dnt.$$

The ratio $\frac{a^2}{r^2}$ can be found by the above theorem, and a substitution and integration will give the equation of centre, or $v - nt$. Bessel objected to this method as not being the simple and natural solution of the question.

In his *Mécanique* Poisson has indicated a method of obtaining these quantities by definite integrals, and this is the method

followed by Bessel. This problem is old, and the coefficients are accurately known. The aim of the writers is to find general expressions for the coefficients, which are complicated for the equation of centre, and I wish to point out that the coefficients can be found directly and easily from the series. Assume with Poisson,

$$r = A_0 + A_1 \cos nt + A_2 \cos 2nt + \ldots + A_i \cos int + \ldots$$
$$v - nt = \qquad B_1 \sin nt + B_2 \sin 2nt + \ldots + B_i \sin int + \ldots$$

If i and i' are positive whole numbers, and different, the integrals of the differentials $\cos int \cos i'ntd \,.\, nt$ and $\sin int \sin i'ntd \,.\, nt$ are zero between the limits v and π. If $i = i'$, the value is $\frac{\pi}{2}$. Hence

$$A_i = \frac{2}{\pi}\int_0^\pi r \cos int \,.\, dnt$$

$$B_i = \frac{2}{\pi}\int_0^\pi (v - nt) \sin int \,.\, dnt$$

For $i = 0$, we have an exception, and

$$A_0 = \frac{1}{\pi}\int_0^\pi r \,.\, dnt = \frac{a}{\pi}\int_0 (1 - e \cos u)^2 \,.\, du = a\left(1 + \frac{e^2}{2}\right)$$

where u is the eccentric anomaly. Since the anomalies have the values 0 and π at the same time, by means of the known values of the differentials of v and nt we change the variable to u, integrate by parts, and omit the second part of B_i, which is a complete differential, and its integral is zero at the limits. Thus we have

$$A_i = -\frac{2ae}{i\pi} \,.\int_0^\pi \sin (iu - ie \sin u) \sin u du$$

$$B_i = \frac{2\sqrt{1 - e^2}}{i\pi} \,.\int_0^\pi \frac{\cos (iu - ie \sin u)\, du}{1 - e \cos u}$$

By expansion

$$\sin (iu - ie \sin u) \sin u du$$

$$= + \sin iu \left(\sin u - \frac{i^2e^2 \sin^3 u}{\lfloor 2} + \frac{i^4e^4 \sin^5 u}{\lfloor 4} - \frac{i^6e^6 \sin^7 u}{\lfloor 6} + \ldots\right) du$$

$$- \cos iu \left(ie \sin^2 u - \frac{i^3e^3 \sin^4 u}{\lfloor 3} + \frac{i^5e^5 \sin^6 u}{\lfloor 5} - \frac{i^7e^7 \sin^8 u}{\lfloor 7} + \ldots\right) du$$

In order to integrate and find the coefficients A_i, we need only a table for converting powers of $\sin u$ into sines and cosines of the multiple arcs. Then by making i equal to 1, 2, 3, 4 ... successively we pick out the term that gives the value $\frac{\pi}{2}$, multi-

plied by a power of e, and a numerical coefficient. The rest of the terms vanish. When i is even, the first line is zero; and when i is odd, the second line is zero. As a trial I have computed in this way the twenty-five terms of the A_i, the coefficients including the ninth power of e. Comparing these, without revision, with Le Verrier's I made only three errors. The work is very simple.

Expanding the differential part of B_i, we have

$$+\left(1-\frac{i^2e^2\sin^2 u}{\lfloor 2}+\frac{i^4e^4\sin^4 u}{\lfloor 4}-\frac{i^6e^6\sin^6 u}{\lfloor 6}+\ldots\right)\cos iu\,.\,du$$

$$+\left(ie\sin u-\frac{i^3e^3\sin^3 u}{\lfloor 3}+\frac{i^5e^5\sin^5 u}{\lfloor 5}-\ldots\right)\sin iu\,.\,du$$

$$+\left(e-\frac{i^2e^3\sin^2 u}{\lfloor 2}+\frac{i^4e^5\sin^4 u}{\lfloor 4}-\frac{i^6e^7\sin^6 u}{\lfloor 6}+\ldots\right)\cos iu\cos u\,.\,du$$

$$+\left(ie^2\sin u-\frac{i^3e^4\sin^3 u}{\lfloor 3}+\frac{i^5e^6\sin^5 u}{\lfloor 5}-\ldots\right)\sin iu\cos u\,.\,du$$

&c. &c. &c.

The first two lines can be integrated as before. For the third line an integration by parts gives the partial value

$$B'_i=\frac{2}{\pi}\,.\,\frac{\sqrt{1-e^2}}{n+1}\,.\int_0^\pi \sin^{n+1}u\sin iu\,.\,du$$

and the fourth line

$$B''_i=-\frac{2}{\pi}\,.\,\frac{\sqrt{1-e^2}}{n+1}\,.\int_0^\pi \sin^{n+1}u\cos iu\,.\,du$$

The lines that follow in the expansion will return to the above forms. n is any exponent of $\sin u$, and for any value of n we make i, 1, 2, 3, 4 . . . &c. Here the work is longer, and we have to multiply by the expanded form of $(1-e^2)^{\frac{1}{2}}$.

Hansen has given some elegant forms for the equation of centre. He puts $e=\sin\phi$, and expands in powers of $\beta=\tan\frac{1}{2}\phi$. His β is the same as the λ of Laplace. The objection of Bessel to the method of Lagrange does not appear to be well founded. It should be a function of the higher analysis to furnish better methods for practical work.

Goshen, Conn.:
1904 *March* 21.

The Rousdon Variable Star Observations.

By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. The observations of long-period variables made during the years 1886–1900 at the Rousdon Observatory, Lyme Regis, Devon, under the direction of the late Sir C. E. Peek, Bart., were put into my hands for discussion and publication after Sir C. E. Peek's death, and will appear as Vol. LV. of the *Memoirs.* A brief account of the discussion may be convenient for other observers.

2. Comparisons were made whenever possible between the variable and *five* stars within a field approximately 1° square. It was of course impossible always to find five stars in this area within a few tenths of a magnitude of the variable; and the differences estimated often exceed a magnitude, sometimes two or three. The accidental errors of such comparisons are naturally large; but there are, nevertheless, distinct advantages in the method which appear on discussion. As regards systematic error it appears that a certain quantity (about 0·2 mag.) must be *numerically subtracted* from all estimated differences greater than about ±0·4 mag. But the value of this quantity changes sensibly with the brightness of the stars compared, and also at two epochs during the work. Still it is not difficult to correct with tolerable accuracy for this arbitrary scale.

3. There is some evidence of systematic error depending on position-angle, but it is conflicting; and, after several vain attempts to establish laws for it from the material available, the quest was abandoned.

4. There seems to be no doubt that the magnitudes for the comparison stars determined at Harvard will not in all cases suit the Rousdon observations. Mr. Grover sees some of them systematically brighter or fainter, doubtless owing to peculiarities of colour. Some trouble was taken to deduce individual magnitudes which would suit his observations, while yet conforming generally to the Harvard scale. But the consequent corrections have been exhibited in a separate column (as also those for the scale value) so that they can be omitted if this is thought desirable.

5. A comparison with the Harvard observations (*Harvard Annals*, vol. xxxvii., Part I.) is given. Two important deductions are made from this comparison.

(*a*) *Systematic Errors.*—The stars observed were divided into two classes, A and B, according to their "redness" as given in Chandler's Third Catalogue. Six stars in Class A (viz. *R*, *S*, and *T Cassiopeiæ*, *S* and *T Cephei*, and *R Aurigæ*) have a mean redness 7·1; and five stars in Class B (viz. *S* and *T Ursæ Majoris*, *S Boötis*, *R Camelopardi*, and *R Draconis*) have a mean redness 2·4. The differences, Rousdon—Harvard, were